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Wave Propagation under Confinement Break

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Abstract:*Propagationof electromagnetic waves has a strong dependence with the refraction index changes. When the media breaksabruptlyhis confinement properties and the evanescent waves turn to be travelling signals, we must adopt a different perspective to understand how are the new rules or which is the instrument to take into account new concepts that are playing an important role like resonances. May be, the natural way to realize this is by means of the Fredholm formulation. But the must used tool for electromagneticwaves are the Maxwell equations. So it would be interesting to give an alternative point of view that remember these last equations but maintain some of the relevant aspectsofthe Fredholm formulation. As we said, the purpose is to describe the transient between free and confinement electromagnetic conditions. That is how we can observe the transition between evanescent and travelling waves. This is the central goal of the present chapter; we develop a new formulation proving that is equivalent to any conventional one. In this way we introduce also new concepts like the Plasma Sandwich Model (PSM) very close to our central task and based in recent experiments for tuning the refraction index.*

Keywords: *Evanescent Waves, Electromagnetic Confinement, Time Reversal, Negative refraction Index.*

I. Introduction

With the aim to analyzethe behavior of the electromagnetic waves we have developed a method that generally we call the Fredholm formulation because is based on using the properties of the integral Fredholm`s equations[2,3,4,5,6,8,13]. Indeed we named generalized Fredholm`s equations (GFE) to a very broad class of integral equations that can be written also in an algebraic form. In this way we have proved for special electromagnetic systems some theorems and obtained useful results but with limited reach, particularly for discrete systems. Nevertheless we could make an approach for many continuous systems because the theoretical basement certainly allows us it in the future. Recalling the initial goal, we give a blueprint of a general procedure for taking into account the Maxwell equations as a guide for obtain new hybrid equations that preserve those properties discovered with Fredholm's formalism.So the hard work will be to show the equivalence between different formalisms, that is the GFE and the new hybrid equations we are going to build. Because in practice we are mixing different kinds of equations we must be very careful to determine how we can apply them. We show how we can apply the new formalism to a common system but introducing the new concepts like resonances, orthogonality and the PSM parameters. Finally we show how the new equations describe the confinement break.

1. Hermitian differential operators and generalized Fredholm's equations

Now we give a glance to both formalismsinvolved, that is the generalized Fredholm'sand the Maxwell equations. Because we are interested on the resonance behaviour we need indeed the homogeneous generalized Fredholm's equations (HGFE), which we can write as [2,3,8]:

$$E^{m}(\bar{r},\omega)=\eta(\omega)\int_{0}^{\infty}\mathrm{K}_{n}^{m(\circ)}(\omega;\bar{r},\bar{r}')E^{n}(\bar{r}',\omega)\,dr' \quad (1)$$

As we have shown [1,2], we can givea general form for the separable kernel

$$\mathrm{K}_{n}^{m(\circ)}(w;\bar{r},\bar{r}') \quad (2)$$

But on this paper we must take for (2) a very special form for the sake of simplicity. In this way we remember that the free Green function $\mathrm{G}_{n}^{m(\circ)}(w;\bar{r},\bar{r}')$ is also the solution of a differential equation of the general form

$$(\mathbf{L}(\bar{r})-\lambda(\omega))\mathrm{G}_{n}^{m(\circ)}(\omega;\bar{r},\bar{r}')=\delta(\bar{r}-\bar{r}') \quad (3)$$

$\mathbf{L}(\bar{r})$ is an appropriate Hermitian differential operator. In equation (3), $\mathrm{G}_{n}^{m(\circ)}(w;\bar{r},\bar{r}')$ represents the electromagnetic field created by a punctual source. In order to make the algebra more clear, let us introducea strong restriction. The later has no physical inconvenient results.

Accordingly with a theorem we have proved [1], equation (1) can by written for discrete systems as:

$$E^{m}(\bar{r}_{i},\omega)=\eta(\omega)\mathbf{A}_{i,j}^{m,n}\mathbf{G}_{n}^{m(\circ)}(\omega;\bar{r}_{i},\bar{r}_{j})E^{n}(\bar{r}_{j},\omega) \quad (4)$$

It is important to underline that even the system does not be discrete; we always can take a discrete subsystem that can be written in the proposed form (4) when the conditions demanded by one related theorem [1] are satisfied.

Suppose we can write for the kernel the form:

$$K_{n}^{m(\circ)}(\omega;\bar{r},\bar{r}')=$$

$$\begin{bmatrix} \mathbf{G}_{n}^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_{1}') & 0 & 0 & \cdots & \ddots & 0 & \vdots & \vdots \\ 0 & \mathbf{G}_{n}^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_{2}') & 0 & \cdots & 0 & \mathbf{G}_{n}^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_{Q-2}') & 0 & 0 \\ 0 & 0 & \mathbf{G}_{n}^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_{3}') & 0 & \cdots & 0 & \mathbf{G}_{n}^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_{Q-1}') & 0 \\ \vdots & \vdots & 0 & \ddots & \cdots & 0 & 0 & \mathbf{G}_{n}^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_{Q}') \end{bmatrix}$$

(5)

In equation (5) we suppose that the interaction matrix

$$\mathbf{A}_{i,j}^{m,n}=\delta(\bar{r}_{i}-\bar{r}_{j}) \quad (6)$$

Have a continuum first sub index and a discrete second one.

Now we return to equation (1), that is the electric field, so we have

$$E^{m}(\bar{r},\omega)=\eta(\omega)\int_{0}^{\infty}\mathrm{K}_{n}^{m(\circ)}(\omega;\bar{r},\bar{r}')E^{n}(\bar{r}',\omega)\,dr' \quad (7)$$

2. Applying Maxwell Equations toFredholm'sformalism

By applying to equation (7) the differential operator

$$rot=\nabla\times \quad (8)$$

We have the equation

$$\nabla\times\mathbf{E}(\bar{r},\omega)=\eta(\omega)\int_{0}^{\infty}\nabla\times\mathbf{K}(\omega;\bar{r},\bar{r}')\mathbf{E}(\bar{r}',\omega)dr' \quad (9)$$

Now, by using Maxwell equations

$$rot\mathbf{E}(\bar{r},\omega)=i\omega\mathbf{B}(\bar{r},\omega) \quad (10)$$

In order to transform equation (9) we give without rigorously proving in this document, the relation:

$$\nabla\times\mathbf{E}(\bar{r},\omega)\delta(\bar{r})=\mathbf{D}(\bar{r})\times\mathbf{E}(\bar{r},\omega)+i\omega\mathbf{B}(\bar{r},\omega) \quad (11)$$

In equation (11)

$$\mathbf{D}(\bar{r})=\delta(y)\delta(z)\mathbf{i}+\delta(x)\delta(z)\mathbf{j}+\delta(x)\delta(y)\mathbf{k} \quad (12)$$

By substituting equations (5), (10) and (11) in equation (9) we have after some algebra:

$$\mathbf{B}(\bar{r},\omega)=$$

$$-i\frac{\eta(\omega)}{\omega}\int_{0}^{\infty}\mathbf{D}(\bar{r})\times\mathbf{E}(\bar{r}',\omega)dr'+\eta(\omega)\int_{0}^{\infty}\mathbf{K}(\omega,\bar{r}')\mathbf{B}(\bar{r}',\omega)dr' \quad (13)$$

But because we have not magnetic monopoles the equation results

$$\mathbf{B}(\bar{r},\omega)=\eta(\omega)\int_{0}^{\infty}\mathbf{K}(\omega,\bar{r}')\mathbf{B}(\bar{r}',\omega)dr' \quad (14)$$

Or

$$B^m(\bar{r},\omega) = \eta(\omega)\int_0^\infty \mathbf{K}_n^m(\omega;\bar{r},\bar{r}')B^n(\bar{r}',\omega)dr' \qquad (15)$$

In equations (14) and (15) the kernel is now:

$$\boldsymbol{K}_n^{m(\circ)}(\omega;\bar{r},\bar{r}') =$$

$$\begin{bmatrix} \mathbf{G}_n^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_1') & 0 & 0 & \cdots & \ddots & 0 & \vdots & \vdots \\ 0 & \mathbf{G}_n^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_2') & 0 & \cdots & 0 & \mathbf{G}_n^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\overline{r_{Q-2}}') & 0 & 0 \\ 0 & 0 & \mathbf{G}_n^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\bar{r}_3') & 0 & \cdots & 0 & \mathbf{G}_n^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\overline{r_{Q-1}}') & 0 \\ \vdots & \vdots & 0 & \ddots & \cdots & 0 & 0 & \mathbf{G}_n^{(\circ)}(\omega;\bar{r},\bar{r}')\delta(\bar{r}'-\overline{r_Q}') \end{bmatrix} \qquad (16)$$

It is evident that equation (15) is the same as equation (1) with $B^m(\bar{r},W) = E^m(\bar{r},W)$. Then, we can see that it is equivalent to use Maxwell equations or the generalized Fredholm equations.

3. The novel Maxwell resembling equations

We have shown that the HGFE can be written in the algebraic form:

$$\overline{\mathcal{E}_e}^m(\omega) = \left[\eta_e(\omega)\mathbf{G}^{(\circ)}(\omega)\mathbf{A}\right]_n^m \overline{\mathcal{E}_e}^n(\omega) \qquad (17)$$

$$\overline{\mathcal{B}_e}^m(\omega) = \left[\boldsymbol{h}_e(\omega)\mathbf{G}^{(\circ)}(\omega)\mathbf{A}\right]_n^m \overline{\mathcal{B}_e}^n(\omega) \qquad (18)$$

We can apply operator $rot = \nabla\times$ to equations (17) and (18) and by using Maxwell equations obtaining:

$$rot\overline{\mathcal{E}_e}(\omega) = -i\omega\boldsymbol{h}_e(\omega)\mathbf{G}^{(\circ)}(\omega)\mathbf{A}\overline{\mathcal{B}_e}(\omega) \qquad (19)$$

$$rot\overline{\mathcal{B}_e}(\omega) = i\omega\mu\varepsilon\eta_e(\omega)\mathbf{G}^{(\circ)}(\omega)\mathbf{A}\overline{\mathcal{E}_e}(\omega) \qquad (20)$$

In terms of the kernel

$$rot\overline{\mathcal{E}_e}(\omega) = -i\omega\boldsymbol{h}_e(\omega)\mathbf{K}^{(\circ)}(\omega)\overline{\mathcal{B}_e}(\omega) \qquad (21)$$

$$rot\overline{\mathcal{B}_e}(\omega) = i\omega\mu\varepsilon\eta_e(\omega)\mathbf{K}^{(\circ)}(\omega)\overline{\mathcal{E}_e}(\omega) \qquad (22)$$

But we know that the eigenvalue can be written

$$\eta_R(\omega) = e^{ih(\omega_R)} \qquad (23)$$

So, equations (21) and (22) become

$$rot\overline{\mathcal{B}_e}(\omega) = i\omega\mu\varepsilon e^{ih(\omega_e)}\mathbf{K}^{(\circ)}(\omega)\overline{\mathcal{E}_e}(\omega) \qquad (24)$$

$$rot\overline{\mathcal{E}_e}(\omega) = -i\omega e^{-ih(\omega_e)}\mathbf{K}^{(\circ)}(\omega)\overline{\mathcal{B}_e}(\omega) \qquad (25)$$

We could consider equations (24) and (25) as the equivalent to the original Maxwell equations (two of them) for transient processes between right-hand and left-hand like-materials broadcasting conditions. We can see that for regions small enough, we can treat the confinement of the electromagnetic field as a problem of changing boundary conditions. The exponent in equations (24) and (25) makes that both equations seems to be conjugated one of the other but we only used the fact that the phase could be different in each one.

4. Boundary conditions andhybrid equations

As we said in chapter 4, we have a pair of equations that we want to see as Maxwell equivalents ones. But really we have combinations of algebraic versions of the HGFE. This means that we have not a boundary conditions problem but a Green function definition problem. Nevertheless we can think that fixing the form of the kernel is the same as to giving the boundary conditions procedure. Keeping in mind that also equations (24) and (25) connects the fields $\overline{\mathcal{E}}$ and $\overline{\mathcal{B}}$ between initial and final points. If we follow this way, we find a set of equations in which we must determine the phases $h(\omega_e)$, $\eta(\omega_e)$ and the refraction index $n=\sqrt{\mu\varepsilon}$ [1,2,4-11,14,15,16,17]. So the problem is the following: we have a region of the space in which a resonant phenomenon occurs, but we don't know the refraction index neither the phases defined above. We can suggest that the kernel (that is the interaction and the free Green function) is known. Then we can use equations (24) and (25) and sketch the fields with a very important difference with the Maxwell equations, that is, the fact that resonant solutions vanishes at the sources sites. This last condition deserves a comment, that is, we must distinguish among a mathematical condition and physical phenomena. Even strictly a resonance cannot exist over thesources (antennas), the complete electromagnetic field is the sum of every component and the vanishing of some of them comes for the condition that a resonance grows from zero to a non zero value starting at the sources. Finally, we can conclude the knowing of the fields and the other undetermined parameters.

5. An academic example

As we said above we can connect the initial field $\overline{\mathcal{E}}$ with the final field $\overline{\mathcal{B}}$ or vice versa but now we are concerned with the changing of broadcasting media conditions. That is we are asking about the solutions when $n=\sqrt{\mu\varepsilon}$ is a negative number. Consider now the academic special case of only two punctual emitters with the kernel given by

$$\mathbf{K}^{(\circ)} = \begin{bmatrix} {}^{1}\mathbf{K}^{(\circ)} & \mathbf{0} \\ \mathbf{0} & {}^{2}\mathbf{K}^{(\circ)} \end{bmatrix} \quad (26)$$

Where

$${}^{1,2}\mathbf{K}^{(\circ)} = \begin{bmatrix} \frac{\sin\left[(\omega-\omega_p)\delta\right]}{(\omega-\omega_p)\delta} & 0 & 0 \\ 0 & \frac{\sin\left[(\omega-\omega_p)\delta\right]}{(\omega-\omega_p)\delta} & 0 \\ 0 & 0 & \frac{\sin\left[(\omega-\omega_p)\delta\right]}{(\omega-\omega_p)\delta} \end{bmatrix}$$

(27)

Suppose that the fields have not z component and that the emitted electric field measured at the two points $\overline{r_1}$ and $\overline{r_2}$ is given by

$$\mathcal{E}_x(\overline{r_1}) = \mathcal{E}_0\cos(\omega_0) \quad (28)$$

$$\mathcal{E}_y(\overline{r_1}) = \mathcal{E}_0\sin(\omega_0) \quad (29)$$

$$\mathcal{E}_x(\overline{r_2}) = \mathcal{E}_0\cos(\omega_0) \quad (30)$$

$$\mathcal{E}_y(\overline{r_2}) = \mathcal{E}_0\sin(\omega_0) \quad (31)$$

Then the magnetic field measured at the two $\overline{r'_1}$ and $\overline{r'_2}$ is given by the expressions:

$$\mathcal{B}_y(\overline{r_1'}) = -i\omega\mu\varepsilon e^{ih}\mathcal{E}_0 z\left(\sin\left[(\omega-\omega_p)\delta\right]\Big/(\omega-\omega_p)\delta\right)\cos(\omega_0) \quad (32)$$

$$\mathcal{B}_x(\overline{r_1}') = i\omega\mu\varepsilon e^{ih}\mathcal{E}_0 z(\sin\left[(\omega-\omega_p)\delta\right] \big/ (\omega-\omega_p)\delta)\sin(\omega_0) \quad (33)$$

$$\mathcal{B}_y(\overline{r_2}') = -i\omega\mu\varepsilon e^{ih}\mathcal{E}_0 z(\sin\left[(\omega-\omega_p)\delta\right] \big/ (\omega-\omega_p)\delta)\cos(\omega_0) \quad (34)$$

$$\mathcal{B}_x(\overline{r_2}') = i\omega\mu\varepsilon e^{ih}\mathcal{E}_0 z(\sin\left[(\omega-\omega_p)\delta\right] \big/ (\omega-\omega_p)\delta)\sin(\omega_0) \quad (35)$$

As we expected, the resonance grows up with z instead diminishes. Now (in a real broadcasting) we can measure the arrived fields for obtaining $m\varepsilon$ and adjusting the phase h. Because of space, we have not written the fields obtained from equation (25). For electromagnetic packaging needs we suppose that z is adjusted to be small enough. Remember that resonance condition is established once we write the HGFE , that is equations (17) and (18).Specially we have shown the application of equations (24) and (25) to an academic problem constituted by the resonant broadcasting through a possibly limited space. Because of the hybrid essence of the new tool we put the boundary conditions by explicitly giving the kernel stated by equations (26) and (27). On this kernel we have incorporated the plasma sandwich model (PSM) parameters d proportional to the plasma layer thickness, and the plasma frequency w_p.

7. A plasma sandwich model (PSM)

Now we are going to explain what kind of object we named the plasma sandwich model or PSM. To this end, we first describe a very important experiment realized by Xiang-ku Kong et al. [7], about the effect of plasma magnetization into the propagation of polarized electromagnetic waves. They have found that it is possible to switch right-hand material conditions into left-hand material conditions by handle the magnetization intensity and the applied voltage at the middle of a device constituted for an iterated arrangement of three plasma layers. They indicate as responsible, the coupling between the electromagnetic polarized waves and the evanescent waves. We differ on their reasoning, but we have proposed that the true mechanism that increases the rotating effect in the polarization angle is the excitation of propagation modes from the evanescent ones. Also, we named precursors the evanescent waves that break their confinement and are converted in propagating waves when the refraction index turns to be negative. This experiment can be now used as a starting platform by defining a "plasma sandwich model" (PSM), that is, we suppose the propagating media in a broadcasting process can be visualized as a series of plasma layers each of them with different magnetization and electric potentials and with very little number of parameters that govern the sign of the refraction index. That is what we named a PSM.

The way followed by Xiang-ku Kong et al.[7], is the transfer matrix [7,14,15], which relate the elliptical polarized beams with the permittivity tensors $\hat{e}_U$ and $\hat{e}_M$. The former is about the un-magnetized layers; and the last for the magnetized one. Instead of the 4×4 matrix they used to describe an incident elliptically polarized beam that propagates along the z axis, we use our own vector-matrix formalism or VMF [4] in the next section. For convenience we suppose that all Fourier components of the electromagnetic beam are associated with a unique value of the permittivity tensor in each layer. The relevant parameters are the magnetization and the electrical potential intensities.

8. The VMF instead the transfer matrix formalism

Now, we can put the parameters appeared in the PSM into the VMF model [4]. To this end, we remember that equation (1) can be written as:

$$\left[\mathbf{1}-\eta_R(\omega)\mathbf{K}^{(\circ)}(\omega)\right]_n^m \overline{w}_R^n(\omega) = 0 \quad (36)$$

Where the kernel $\mathbf{K}^{(\circ)}(\omega)$ is the product of the free Green function $\mathbf{G}^{(\circ)}(\omega)$ with the interaction $\mathbf{A}$ so explicitly

$$\left[\mathbf{1}-\eta_R(\omega)\mathbf{G}^{(\circ)}(\omega)\mathbf{A}\right]_n^m \overline{w}_R^n(\omega)=0 \quad (37)$$

It is not difficult to think that if we have a transfer matrix description of a problem we must have a VMF version of it. Of course there are very important differences between these formalisms, for example, VMF includes explicitly a mechanism to makes easy a time reversal process. Also we have a frequency domain instead a time dependent one, the former the appropriate for information theory applications. But possibly the most important difference is that VMF formalism includes the concept of the resonant solutions. In the present work, we start with the appropriate VMF generic version of the PSM and then introduce the relevant parameters in the next section.

9. The kernel for a resonance regime

In this section we find the resonant frequencies for a specific problem, but we must remember that those resonant frequencies can be used only to associate an interval of frequencies of a real signal to a device that could be an antenna. The form of the kernel depends of the response of the media in some circumstances that can vary even from a different time interval. So we use an example that is very easy to work but that is not important how is the shape of the signal we used to get it. Now, we can find the resonant frequencies in this academic example. To this end we choose a convenient kernel $\mathbf{K}^{(\circ)}(\omega)$, for simplicity we do not take into accounts the three components of the electromagnetic field. Supposing we only has one, but we have two emitting antennas. A possible kernel is [4]:

$$\mathbf{K}^{(\circ)}(\omega)=\begin{pmatrix} \dfrac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} & -i\dfrac{\cos(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} \\ i\dfrac{\cos(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} & \dfrac{\sin(\omega-\omega_p)\delta}{(\omega-\omega_p)\delta} \end{pmatrix}$$

(38)

In equation (38) we have introduced the PSMparameter d.Explicitly:

$$\delta=\kappa\overline{d}_M \quad (39)$$

In definition (39) κ has the physical meaning of the wave number of an incident beam that interacts with the magnetic and electric fields in a way that the whole kernel is the expressed in equation (38). But $\overline{d}_M$ is an average thickness of a plasma-magnetized layer that generates this interaction. The parameter ω_p is an average value for the plasma frequency in the same magnetized plasma layer that can be expressed in terms of the local electron concentration in the layer as:

$$\omega_p=\frac{1}{2\pi}\left(\frac{Ne^2}{m\varepsilon_0}\right)^{\frac{1}{2}} \quad (40)$$

In this equation N is the electron concentration, ε_0 is the permittivity of vacuum and e the electronic charge.We can observe that the change in these parameters gives different broadcasting regimes. The PSM also supposes that we have not a stationary and unique set of iterated layers but a series of sets changing with time and therefore with different effects for distinct frequencies. At this point, it is important to remember that the equation we must solve is equation (36) where

$$\mathbf{K}_{i,j,m}^{n(\circ)}(\omega)=\begin{Bmatrix} 0 \;\; if \;\; i=j \\ A_j^{n,m}G_\omega^{n,m(\circ)}(\overline{r}_i,\overline{r}_j)\; if \;\; i\neq j \end{Bmatrix} \quad (41)$$

The conditions for resonances are that Fredholm’s determinant for the equation (38) equals zero, and that Fredholm´s eigenvalue λ equals to one [3,8].

These two conditions give us the resonant frequencies for the system constituted by these two antennas but dependent on the plasma sandwich model parameters. Now, we must remember that resonances have a special behavior that can be represented by a complex frequency:

$\omega = \mathrm{K} - i\Lambda$ (42)

The imaginary part Λ is responsible for the amazing transformation of the evanescent waves for travelling ones. Also we have the relation between ω and the wave number κ, that is

$\kappa = \sqrt{\mu\varepsilon}\omega$ (43)

(By substituting (42) and (43) into equation (38) we have that resonance condition can be written as:

$$\Delta\begin{pmatrix} A & -B \\ B & A \end{pmatrix} = 0 \quad (44)$$

II. Concluding Remarks

Our expected goal has been reached, that is we have shown that it is equivalent to employ the HGFE formulation or to use the Maxwell equations. But we also have shown that the natural way to introduce the resonant behaviour is the former. Nevertheless we have found a new tool appropriate for electromagnetic confinement or packaging created from the simultaneous application of both formulations that is the expressed in equations (19) to (25). Specially we have shown the application of equations (24) and (25) to an academic problem constituted by the resonant broadcasting through a possibly limited space. Because of the hybrid essence of the new tool we put the boundary conditions by explicitly giving the kernel stated by equations (26) and (27). On this kernel we have incorporated the plasma sandwich model (PSM) parameters d proportional to the plasma layer thickness, and the plasma frequency w_p.

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